Grade Level/Course: Algebra 1/Algebra 2
Lesson/Unit Plan Name: Solve Exponential Equations
Rationale/Lesson Abstract: This lesson will enable students to solve exponential equations by changing bases and using the property of equality of exponential functions.
Timeframe: Depending on your students' need, this lesson can take 1 or 2 1-hour periods. The break-down is as follows: • Scaffolding (pre-lesson) parts 1 and 2: 30 minutes • Main lesson (including exit ticket and warm-up): 50 minutes • Challenge questions: 15 minutes • Matching activity: 20-30 minutes
Common Core Standard(s): A.SSE.3c: Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Instructional Resources/Materials: Copies of the warm-up, pencil, document camera (to display student work), students in partners (or easily able to move to partners), cut out and grouped copies of the matching activity (one per pair of students, or one per group), and copies of the exit ticket.

Warm-Up

1. Write 2•2•2•2 in exponential form.

2. What is one way to represent 27 in exponential form?

3. Given that $x \neq 0$ and $z \neq 0$, choose Yes or No to indicate which of the following expressions are equivalent to $\frac{4x^5}{z^2}$.

A) $\left(\frac{2x^3}{z}\right)^2$ ◯ Yes ◯ No

B) $\frac{\left(2x^3\right)^3 z^2}{2x^4 z^4}$ ◯ Yes ◯ No

C) $\frac{16x^7 z}{4x^2 z^3}$ ◯ Yes ◯ No

Warm-Up Solutions

1. 2•2•2•2 = 2^4

2. $27 = 3^3$ This is not the only way, but will probably be the most common.

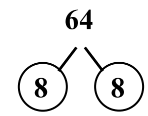
3. **A)** No **B)** Yes **C)** Yes

Activity/Lesson:

Scaffolding Part 1 (pre-lesson):

Depending on your students' mastery of the decomposition of large numbers, you may want to spend some time at the beginning of the class period rewriting large numbers with different bases.

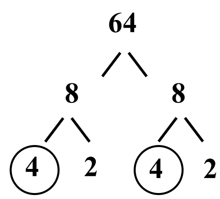
"Let's write 64 in base 8. We are looking for factors of 8 in 64. How many factors of 8 are there in 64?" **You can use a factor tree to decompose 64**



$\therefore 64 = 8^2$

We do not have to continue to decompose 8 because we are looking for factors of 8.

"Let's write 64 in base 4. We are looking for factors of 4 in 64. How many factors of 4 are there in 64?" **Use another factor tree. Stop decomposing when you see a factor of 4.**

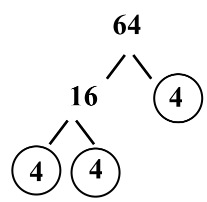


$64 = 4^2 \bullet 2 \bullet 2$

$= 4^2 \bullet 4$

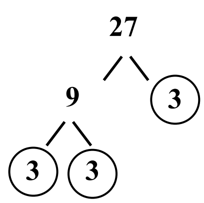
$= 4^3$

OR



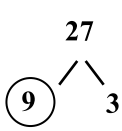
$\therefore 64 = 4^3$

"Let's write 27 in base 3."



$\therefore 27 = 3^3$

"Let's write 27 in base 9."



We can't write 27 in base 9 because 27 only has one factor of 9 and another factor of 3. We cannot write 27 using ONLY factors of 9, therefore we can't write 27 in base 9 with a whole number exponent.

NOTE: $27 = 9^{\frac{3}{2}}$

For more practice, use other examples of large numbers that students will see during the remainder of the lesson. (ex. 81, 16, 8, 32, 25, 125). Be sure to emphasize in which base each number CAN and CANNOT be re-written.

Scaffolding Part 2 (pre-lesson):

Depending on your students' mastery of exponents, you may want to spend the beginning of the lesson reviewing exponent properties. The property they MUST have mastered to succeed in this lesson is the Power of a Power Property, which states: $\left(x^a\right)^b = x^{a \cdot b}$

Property of Equality of Exponential Functions:

If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$. In other words, if the bases are the same, then the exponents must be equal.

Example 1: Solve.

$12 = 12^x$	← The bases are exactly the same
$12^1 = 12^x$	$12 = 12^1$
$\therefore \quad 1 = x$	Property of Equality of Exponential Functions

Example 2: Solve.

$8^{x-3} = 8^4$	← The bases are exactly the same
$\therefore \quad x - 3 = 4$	Property of Equality of Exponentials
$x - 3 + 3 = 4 + 3$	Inverse operations
$x = 7$	

Example 2: CHECK SOLUTION

$8^{x-3} = 8^4$	
$8^{7-3} = 8^4$	Substutite our answer, $x = 7$
$8^4 = 8^4$	Check!

You Try: Solve.

1. $100^6 = 100^x$

2. $5^{2x} = 5^3$

3. $2^{y-1} = 2^{-10}$

Solutions to You Tries:

1. $100^6 = 100^x$

$\therefore \quad 6 = x$

2. $5^{2x} = 5^3$

$\therefore \quad 2x = 3$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$

3. $2^{y-1} = 2^{-10}$

$\therefore \quad y - 1 = -10$

$y - 1 + 1 = -10 + 1$

$y = -9$

Example 3: Solve

$$5^x = 25$$

(Change the base ↓)

$$5^x = 5^2$$

$$\therefore \quad x = 2$$

Teacher Talk (Think Aloud)

How can we solve this problem? We cannot divide both sides by 5, that would be a problem like $5x = 25$. We cannot take the square root, that would be a problem like $x^2 = 25$. To solve this, we are going to use the Property of Equality of Exponential Functions.

Are the bases the same here? *No*

I would like to change 25 to be written in base 5.

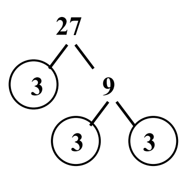
I can write 25 in base 5 because $5 \bullet 5 = 5^2 = 25$.

Now, the bases are the same and I can use the Property of Equality.

Example 4: Solve

If students are having difficulty recognizing that $27 = 3^3$, ask, "How many 3s must we multiply to get to 27?"

You could also show a factor tree:



There are 3 factors of 3 in 27, therefore $27 = 3^3$

$$27 = 3^{4x}$$

(Change the base ↓)

$$3^3 = 3^{4x}$$

$$\therefore \quad 3 = 4x$$

$$\frac{3}{4} = \frac{4x}{4}$$

$$x = \frac{3}{4}$$

Teacher Talk with Student Help (We Do)

Are the bases the same here? *No.*

What base do I want on both sides of the equation? *Base 3.*

Why? *Because the right side of the equation already has base 3 and, if we'd like to use the Property of Equality, we need equal bases.*

Can we change the left side of the equation to base 3? *Yes*

As we went over in the last Warm-Up question, we can write 27 as 3^3.

Now the bases are the same and I can use the Property of Equality.

Think-Pair-Share:

What are the differences between $3^x = 81$ and $9^y = 81$? Would you solve these differently? Solve each problem.

Give students time to think about this problem on their own, then instruct them to discuss with a partner. After they have shared with their partner, call on a few pairs to share their findings with the class and show their work under the document camera. This should emphasize that you may change the same number (81) into different bases ($81 = 3^4 = 9^2$) depending on the needs of the particular problem.

You Try: Solve.

4. $8 = 2^x$ **5.** $4^{x+1} = 64$ **6.** $64 = 8^{2x}$

Solutions to You Tries:

4. $8 = 2^x$

$2^3 = 2^x$

$\therefore \; 3 = x$

5. $4^{x+1} = 64$

$4^{x+1} = 4^3$

$\therefore \; x + 1 = 3$

$x + 1 - 1 = 3 - 1$

$x = 2$

6. $64 = 8^{2x}$

$8^2 = 8^{2x}$

$\therefore \; 2 = 2x$

$\frac{2}{2} = \frac{2x}{2}$

$1 = x$

Point out that in #5 you want to change 64 to base 4 ($64 = 4^3$), but in #6 you need to change 64 to base 8 ($64 = 8^2$).

Example 5: Solve

$9^{3y} = 27$

Teacher Talk with Student Help

Are the bases the same here? *No.*

What base do I want to change 27 to? *Base 9.*

Would it be easy to change 27 to base 9? *No.*

Why not? *Because there is no integer exponent for 9 that will equal 27.*

So we'll have to change bases on BOTH sides of the equation.

What base can I change 9 into? *Base 3.*

Can I also change 27 to base 3? *Yes.*

(Change the base ↓   Change the base ↓)

$\left(3^2\right)^{3y} = 3^3$	Change bases
$3^{2 \bullet 3y} = 3^3$	Power of a Power Property
$\therefore \; 2 \bullet 3y = 3$	Property of Equality of Exponential Functions
$6y = 3$	Simplify
$\frac{6y}{6} = \frac{3}{6}$	Inverse operations
$y = \frac{1}{2}$	Simplify

Example 6: Solve

$$32 = 4^{x-3}$$

Change the base | Change the base

Teacher Talk with Student Help

Are the bases the same here? *No.*

What base do I want to change 32 to? *Base 4.*

Would it be easy to change 32 to base 4? *No.*

Why not? *Because there is no integer exponent for 4 that will equal 32.*

So we'll have to change bases on BOTH sides of the equation.

What base can I change 4 into? *Base 2.*

Can I also change 32 to base 2? *Yes.*

$2^5 = \left(2^2\right)^{x-3}$	Change bases
$2^5 = 2^{2(x-3)}$	Power of a Power Property
$\therefore\ 5 = 2(x-3)$	Property of Equality of Exponential Functions
$5 = 2x - 6$	Distribute
$5 + 6 = 2x - 6 + 6$	Inverse operations
$11 = 2x$	Simplify
$\frac{11}{2} = \frac{2x}{2}$	Inverse operations
$\frac{11}{2} = x$	

You Try: Solve.

7. $16^x = 8$ **8.** $125 = 25^{6y}$ **9.** $9^{x-1} = 27$

Solutions to You Tries:

7. $16^x = 8$

$$\left(2^4\right)^x = 2^3$$
$$2^{4 \cdot x} = 2^3$$
$$\therefore\ 4x = 3$$
$$\frac{4x}{4} = \frac{3}{4}$$
$$x = \frac{3}{4}$$

8. $125 = 25^{6y}$

$$5^3 = \left(5^2\right)^{6y}$$
$$5^3 = 5^{2 \cdot 6y}$$
$$\therefore\ 3 = 12y$$
$$\frac{3}{12} = \frac{12y}{12}$$
$$\frac{1}{4} = y$$

9. $9^{x-1} = 27$

$$\left(3^2\right)^{x-1} = 3^3$$
$$3^{2(x-1)} = 3^3$$
$$\therefore\ 2(x-1) = 3$$
$$2x - 2 = 3$$
$$2x - 2 + 2 = 3 + 2$$
$$2x = 5$$
$$\frac{2x}{2} = \frac{5}{2}$$
$$x = \frac{5}{2}$$

Have students come up to the document camera to show their work for the You Tries. Have them explain and justify each step.

Challenge Problems:

1. Solve $9^{2x} = 27^{x-3}$

2. Solve $125^{2x-1} = 25^{4x}$

3. Solve $4^{-2x} = 8^{-x+1}$

Solutions to Challenge Problems:

1.

$$\begin{aligned} 9^{2x} &= 27^{x-3} \\ \left(3^2\right)^{2x} &= \left(3^3\right)^{x-3} \\ 3^{2\cdot 2x} &= 3^{3(x-3)} \\ \therefore\quad 4x &= 3(x-3) \\ 4x &= 3x-9 \\ 4x-3x &= 3x-3x-9 \\ x &= -9 \end{aligned}$$

2.

$$\begin{aligned} 125^{2x-1} &= 25^{4x} \\ \left(5^3\right)^{2x-1} &= \left(5^2\right)^{4x} \\ 5^{3(2x-1)} &= 5^{2\cdot 4x} \\ \therefore\quad 3(2x-1) &= 8x \\ 6x-3 &= 8x \\ 6x-6x-3 &= 8x-6x \\ -3 &= 2x \\ \frac{-3}{2} &= \frac{2x}{2} \\ -\frac{3}{2} &= x \end{aligned}$$

3.

$$\begin{aligned} 4^{-2x} &= 8^{-x+1} \\ \left(2^2\right)^{-2x} &= \left(2^3\right)^{-x+1} \\ 2^{2(-2x)} &= 2^{3(-x+1)} \\ \therefore\quad -4x &= 3(-x+1) \\ -4x &= -3x+3 \\ -4x+3x &= -3x+3x+3 \\ -x &= 3 \\ \frac{-x}{-1} &= \frac{3}{-1} \\ x &= -3 \end{aligned}$$

Matching Activity: The cards are on page 9 ready to be copied and cut out. Give each pair or group of students a set of the cards.

Tell students to **sort the cards into 3 piles by whichever characteristics they choose.** Then, have groups come up to share how they sorted the cards. Guide students (if they haven't already) to sort in to groups of: problems with different bases, problems with the same base, and solutions.

Give students 10 minutes to **work out all the problems** (showing their work on a separate sheet of paper) and match the cards into 7 sets. When they are finished, check their work and (if time allows) have students come up to the document camera to show how they completed each problem and why they matched their sets in the way they did.

Assessment: The Exit Ticket is on page 10 in student-friendly format. Here are the solutions:

1. Solve $4^{2x} = 16$

$$\begin{aligned} 4^{2x} &= 4^2 \\ \therefore\quad 2x &= 2 \\ \frac{2x}{2} &= \frac{2}{2} \\ x &= 1 \end{aligned}$$

OR

$$\begin{aligned} \left(2^2\right)^{2x} &= 2^4 \\ 2^{2\cdot 2x} &= 2^4 \\ \therefore\quad 4x &= 4 \\ \frac{4x}{4} &= \frac{4}{4} \\ x &= 1 \end{aligned}$$

2. Solve $81 = 27^{x+1}$

$$\begin{aligned} 3^4 &= \left(3^3\right)^{x+1} \\ 3^4 &= 3^{3(x+1)} \\ \therefore\quad 4 &= 3(x+1) \\ 4 &= 3x+3 \\ 4-3 &= 3x+3-3 \\ 1 &= 3x \\ \frac{1}{3} &= x \end{aligned}$$

$16^{2x} = 8$	$\left(2^4\right)^{2x} = 2^3$	$x = \frac{3}{8}$
$16^{2x} = 4$	$\left(2^4\right)^{2x} = 2^2$	$x = \frac{1}{4}$
$8^{2x} = 4$	$\left(2^3\right)^{2x} = 2^2$	$x = \frac{1}{3}$
$64 = 4^{2x+1}$	$4^3 = 4^{2x+1}$	$x = 1$
$32 = 4^{2x+1}$	$2^5 = \left(2^2\right)^{2x+1}$	$x = \frac{3}{4}$
$9^{x+4} = 27$	$\left(3^2\right)^{x+4} = 3^3$	$x = -\frac{5}{2}$
$3^{x+4} = 27$	$3^{x+4} = 3^3$	$x = -1$

Exit Ticket

Name: ______________________________________ Date: ___________ Period/Block: _____

1. Solve $4^{2x} = 16$

2. Solve $81 = 27^{x+1}$

Exit Ticket

Name: ______________________________________ Date: ___________ Period/Block: _____

1. Solve $4^{2x} = 16$

2. Solve $81 = 27^{x+1}$

Exit Ticket

Name: ______________________________________ Date: ___________ Period/Block: _____

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